OBERSEMINAR DARSTELLUNGSTHEORIEGRUPPE SOSE 2018: CLUSTER ALGEBRAS

1. Mutations and cluster algebras of geometric type

References: [Mar13, Chapters 2.1-2.3], [FZ02, Chapter 4], [Kel]

- Give the definitions of a seed, mutation of a seed and cluster algebra of geometric type.
- Provide examples.
- State the facts about skew-symmetric matrices (as in [Mar13, Chapter 2.2]) and give at least the idea of the proofs as can be found in [FZ02, Chapter 4].
- Introduce the quiver notation and mutation of quivers. Provide examples of quiver mutation using Keller's quiver mutation applet [Kel].

2. Exchange pattern cluster algebras

Reference: [Mar13, Chapter 3]

- Define exchange patterns and exchange pattern cluster algebras.
- Give the characterization of exchange patterns by sign-skew-symmetric matrices [Mar13, Proposition 3.3.4].
- Exchange patterns of geometric type.
- Describe the connection between cluster algebras of geometric type and exchange pattern cluster algebras [Mar13, Theorem 3.3.9].

3. Reflection groups

Reference: [Mar13, Chapter 4]

- Shortly review the necessary background on reflection groups, (crystallographic) root systems, Coxeter groups and the classification of finite reflection groups resp. Coxeter groups [Mar13, Chapters 4.1-4.6, 4.9].
- reduced expressions, longest element, exponents, Coxeter elements, Theorem of Chevalley [Mar13, Chapters 4.10+4.11].

4. Cluster algebras of finite type

Reference: [Mar13, Chapter 5], [BGZ06]

- Give the definitions of (strongly) isomorphic cluster algebras, cluster algebras of finite type and Cartan counterpart [Mar13, Chapters 2.5+5.1].
- Define valued quiver [Mar13, Chapter 2.4].
- State the classification results for cluster algebras of finite type: [Mar13, Theorems 5.12 +5.13] and [BGZ06, Theorems 1.1+1.2] (the latter ones should be stated as one theorem).
- State the bijection between almost positive roots and cluster variables [Mar13, Theorem 5.3.1], Q-root cluster.
- [Mar13, Theorem 5.3.4]

- State the formula for the number of seeds for a cluster algebra of Dynkin type [Mar13, Chapter 5.6].

5. Cluster fan and cluster complex

Reference: [Mar13, Chapter 6]

- Define Q-root cluster and provide the necessary background [Mar13, Chapters 5.4+5.5].
- Introduce the cluster fan, i.e [Mar13, Theorem 6.2.3] and the essential ideas and steps of the proof.
- Introduce the cluster complex [Mar13, Chapter 6.3].

6. Generalized Associahedra

Reference: [Mar13, Chapter 6]

- State the theorem about the realization of the generalized associahedron via the cluster fan of a Dynkin quiver [Mar13, Theorem 6.5.1].
- Provide background and examples: [Mar13, Chapter 6.6] and [FR07, Chapters 4.3-4.5].

7. The Laurent Phenomenon

References: [Mar13, Chapter 7], [FZ02, Chapter 3], [Lam13, Chapter 4]

The aim of this talk should be to state and proof the Laurent Phenomenon for cluster algebras. A good reference for an accessible proof might be [Lam13].

8. Grassmannians

Reference: [Mar13, Chapter 9]

Show that the homogeneous coordinate ring of the Grassmannian of k-subspaces of an n-dimensional space can be regarded as a cluster algebra.

References

[BGZ06] M. Barot, C. Geiss, A. Zelevinsky, Cluster algebras of finite type and positive symmetrizable matrices, *J. London Math. Soc. (2)* **73** (2006), 545-564.

[BM15] M. Barot and R.J. Marsh, Reflection group presentations arising from cluster algebras, *Trans. Amer. Math. Soc.* **367** (2015), 1945-1967.

[FR07] S. Fomin, N. Reading, Root systems and generalized associahedra, *Geometric combinatorics*, 63-131, IAS/Park City Math. Ser., 13, Amer. Math. Soc., Providence, RI, 2007

[FST08] S. Fomin, M. Shapiro, D. Thurston, Cluster algebras and triangulated surfaces. Part I: Cluster complexes, *Acta Math.* **201** (2008), 83-146.

[FZ02] S. Fomin, A. Zelevinsky, Cluster algebras I: Foundations, *Journal of the AMS* **15** (2002), 497-529.

[FZ03] S. Fomin, A. Zelevinsky. Cluster algebras II: Finite type classification, *Invent. Math.* **154** (2003), 63-121.

[Kel] B. Keller, Quiver mutation in Java, `http://people.math.jussieu.fr/keller/quivermutation`

[Lam13] P. Lampe, lecture notes for a course on cluster algebras, Bielefeld (2013). `https://www.math.uni-bielefeld.de/~lampe/teaching/cluster/cluster.pdf`

[Mar13] R. J. Marsh, *Lecture Notes on Cluster Algebras*, Zürich Lectures in Advanced Mathematics, EMS (2013).