

Übungen zu Classical Groups — Blatt 5

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Woche: 9+10

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Throughout K is a field of characteristic $\neq 2$ and (V, Q) is quadratic space over K .

Exercise 17: For $0 \neq u \in V$ isotropic and $y \in \langle u \rangle^\perp$ let $\rho_{u,y} \in \text{GO}(V)$ be the Siegel transformation. Let $v \in V$ such that (u, v) is a hyperbolic pair. Show that we may assume $y \in \langle u, v \rangle^\perp$, and then we have $\rho_{u,y}(v) = v - \frac{1}{2}Q(y)u - y$.

Exercise 18: Work out the details for the proof that $\Omega(V)$ acts primitively on $\mathbb{P}\mathcal{C}$.

Exercise 19: Let V be a quadratic hyperbolic plane, with hyperbolic pair (u, v) . Show that $M^{2,2}(K)$ is compatible with Q via

$$\varphi : V \rightarrow M^{2 \times 2}(K), \quad u \mapsto \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad v \mapsto \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$

and that $\hat{\varphi} : C(V) \rightarrow M^{2 \times 2}(K)$ is an isomorphism.

Exercise 20: Show that the Clifford algebra of a quadratic space V is unique up to K -algebra isomorphism.