Übungen zu Classical Groups — Blatt 4

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Throughout K is a field and V is a finite-dimensional K-vector space.

Exercise 13: Let V be a quadratic space. Show the following:

(a) If $u \in V$ is anisotropic and $g \in \mathrm{GO}(V)$ then ${}^g\sigma_u = \sigma_{g.u}$.

(b) Any hyperbolic plane in V contains exactly two totally isotropic lines.

(c) If H is a hyperbolic plane and $g \in \mathrm{GO}(H)$ fixes an isotropic vector $0 \neq v \in H$ then $g = \mathrm{id}_H$.

Exercise 14: (a) Determine the Witt group of $\mathbb{F}_q$, for q an odd prime power.

(b) Determine the Witt group of a quadratically closed field (e.g., $\mathbb{C}$).

(c) Determine the Witt group of $\mathbb{R}$.

(d) Show there are infinitely many inequivalent quadratic spaces of dimension 2 over $\mathbb{Q}$.

[**Hint**: First show that equivalent forms represent the same values.]

Exercise 15: Prove the Witt extension theorem for symplectic spaces.

Exercise 16: Let V be a quadratic space of dimension n and Witt index $m > 0$ over $\mathbb{F}_q$. Show that the number of totally isotropic subspaces of dimension k in V is

$$\prod_{i=0}^{k-1} \frac{q^m - q^i}{q^k - q^i} \cdot \prod_{i=0}^{k-1} (q^{m-e-i} + 1),$$

where $e := 2m - n + 1$.