

## Übungen zu Classical Groups — Blatt 1

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Woche: 1+2

WS 22/23

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Throughout  $K$  is a field and  $V$  is a finite-dimensional  $K$ -vector space.

**Exercise 1:** Show the following:

- (a)  $Z(\mathrm{GL}(V)) = \{a \mathrm{id}_V \mid a \in K^\times\}$ , and  $Z(\mathrm{SL}(V)) = \{a \mathrm{id}_V \mid a \in K^\times, a^n = 1\}$ .
- (b)  $\mathrm{GL}(V)$  naturally acts on  $\mathbb{P}(V)$  by  $g \cdot [v] := [g \cdot v]$  for  $g \in \mathrm{GL}(V)$ ,  $0 \neq v \in V$ , and the kernel of this action is  $Z(\mathrm{GL}(V))$ .

**Exercise 2:** Show the following:

- (a) The inverse of a transvection is a transvection.
- (b) Let  $V$  be a subspace of  $V_1$ ,  $v \in V_1 \setminus V$ , and  $t \in \mathrm{GL}(V)$  a transvection. Then  $t$  can be extended to a transvection  $t_1 \in \mathrm{GL}(V_1)$  with fixed hyperplane containing  $v$ .

**Exercise 3:** Let  $G$  be a transitive permutation group on  $X$ . Then  $G$  is primitive on  $X$  if and only if the stabilizer  $G_x$  of any  $x \in X$  is a maximal subgroup of  $G$ .

**Exercise 4:** Show  $\mathrm{GL}(V)' = \mathrm{SL}(V)$  when  $\dim V \geq 3$ , or  $\dim V = 2$  and  $|K| > 3$ . What happens in the excluded cases?